



**University
of Victoria**

Graduate Studies

Notice of the Final Oral Examination
for the Degree of Doctor of Philosophy

of

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MSc (University of Victoria, 2013)
BSc (Beijing Institute of Technology, 2011)

“Theory and Algorithms for Finding Optimal Regression Design”

Department of Mathematics and Statistics

Tuesday, June 20, 2017

1:00 P.M.

David Turpin Building
Room A136

Supervisory Committee:

Dr. Julie Zhou, Department of Mathematics and Statistics, University of Victoria (Supervisor)
Dr. Boualem Khouider, Department of Mathematics and Statistics, UVic (Member)
Dr. Farouk Nathoo, Department of Mathematics and Statistics, UVic (Member)
Dr. Stan Dosso, School of Earth and Ocean Sciences, UVic (Outside Member)

External Examiner:

Dr. Zhide Fang, Department of Biostatistics, Louisiana State University

Chair of Oral Examination:

Dr. Judith Clarke, Department of Economics, UVic

Dr. David Capson, Dean, Faculty of Graduate Studies

Abstract

In this dissertation, we investigate various theoretical properties of optimal regression designs and develop several numerical algorithms for computing optimal regression designs. The results can be applied to linear, nonlinear and generalized linear models.

Our work starts from how to solve the design problems for A-, As, c-, I- and L-optimality criteria on one-response model. Theoretical results are hard to derive for many regression models and criteria, and existing numerical algorithms can not compute the results efficiently when the number of support points is large. Therefore we consider to solve the design problems based on SeDuMi program in MATLAB. SeDuMi is developed to solve semidefinite programming (SDP) problems in optimization. To apply it, we derive a general transformation to connect the design problems with SDP problems, and propose a numerical algorithm based on SeDuMi to solve these SDP problems. The algorithm is quite general under the least squares estimator (LSE) and weighted least squares estimator (WLSE) and can be applied to both linear and nonlinear regression models.

We continue to study the optimal designs based on one-response model when the error distribution is asymmetric. Since the second-order least squares estimator (SLSE) is more efficient than the LSE when the error distribution is not symmetric, we study optimal designs under the SLSE. We derive expressions to characterize A- and D-optimality criteria and develop a numerical algorithm for finding optimal designs under the SLSE based on SeDuMi and CVX programs in MATLAB. Several theoretical properties are also derived for optimal designs under SLSE. To check the optimality of the numerical results, we obtain the Kiefer-Wolfowitz equivalence theorem and apply it to various applications.

Finally, we discuss the optimal design problems for multi-response models. Our algorithm studied here is based on SeDuMi and CVX, and it can be used for linear, nonlinear and generalized linear models. The transformation invariance property and dependence on the covariance matrix of the correlated errors are derived. We also correct the errors in the literature caused by formulation issues.

The results are very useful to construct optimal regression designs on discrete design space. They can be applied for any one-response and multi-response models, various optimality criteria, and several estimators including LSE, maximum likelihood estimator, best linear unbiased estimator, SLSE and WLSE.